Surface Current Measurements with an Electric Probe*

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Summary—The use of an electric probe for measurement of induced surface currents on obstacles in a parallel plate region is described. An analysis of the sources of error, and in particular the interaction of the probe with the obstacle, is examined theoretically and experimentally. It is concluded that the technique is capable of yielding measurements of good accuracy.

Introduction

MEASUREMENTS of electromagnetic fields diffracted by cylindrical obstacles, when $\boldsymbol{E}$ is parallel to the axis, are carried out most conveniently and accurately, in a parallel-plane device.[1] This report considers a new technique for measuring surface currents induced on cylindrical obstacles in such a device. In this method an electric unipole probe is used to measure the rate of change of electric field, hence magnetic field, from which the surface current may be determined. In contrast to existing techniques involving loops[2] or slits[3] the unipole probe is simple and readily available.

Probe Technique

Fig. 1 shows the cross section of an arbitrarily curved cylindrical reflecting body. Let the direction of the tangent at an arbitrary point P be x, and choose z to be the axial coordinate; then the surface current J_z is given by .

$$J_z = -H_x = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} \tag{1}$$

where all quantities are evaluated at the origin.

The probe technique for surface current measurement involves, simply, an approximation to $\partial E_z/\partial y$. That is, E_z is measured at a small distance Δy from the conductor and since $E_z = 0$ at the conductor

$$\frac{\partial E_z}{\partial y} \approx \frac{E_z(\Delta y)}{\Delta y}$$

Consequently

$$J_z = CE_z(\Delta y) \tag{2}$$

where $E_z(\Delta y)$ is always measured a fixed distance from the reflector, in which case $C = 1/j\omega\mu\Delta y$ is a constant.

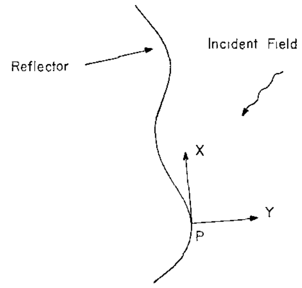

Fig. 1—Cylindrical geometry.

The accuracy of this method is limited, fundamentally, by the approximation of

$$\frac{\partial E_z}{\partial y} \text{ by } \frac{E_z(\Delta y)}{\Delta y}.$$

It is impossible to predict, in advance, what this error will be for an arbitrary shaped reflector. Some guidance can be found by considering the problem of an obliquely incident plane wave on an infinite plane reflector. If the reflector lies in the xz plane and the incident plane wave (parallel polarization) makes an angle θ with y, then the per cent error in calculating J_z from (2) comes out

$$\text{per cent error} = \tfrac{1}{6}(k\Delta y \cos\theta)^2 \times 100 \qquad (k\Delta y = (2\pi/\lambda)\Delta y \ll 1). \tag{3}$$

Thus with probe-reflector spacings of around $\lambda/15$ conventional accuracy may be achieved. Of course, all comparable measurement techniques suffer this same field integrating effect.

Analysis of Probe Interaction

An error of potentially greater seriousness stems from the interaction of the probe and reflector, particularly since their separation is necessarily small. If relative values of surface current is desired then it is important only to ensure that the interaction not change (substantially) with probe position. However, it is easier to discuss the more stringent condition that the interaction itself be small. In formulating the problem we will assume that the reflector may be replaced by an infinite (tangent) plane so that the effect of the reflector may be accounted for by image theory. We shall also include the image of the probe in the parallel-plane sheet which it penetrates. Since short probes are anticipated other images are further away and should not affect the general results significantly; accordingly they are ignored. The model actually corresponds to the case of a unipole over a ground plane, near an infinite plane reflector.

The analytical work is based on an application of the

* Received by the PGMTT, November 20, 1961; revised manuscript received, February 8, 1962. The work presented in this paper was sponsored by the U. S. Air Force Cambridge Research Center on Contract AF 19(604)-3887.

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[1] R. V. Row, "Microwave diffraction measurements in a parallel plate region," *J. Appl. Phys.*, vol. 24, pp. 1448–1452; 1953.

[2] R. King, "The Loop Antenna as a Probe in Arbitrary Electromagnetic Fields," Cruft Lab., Harvard University, Cambridge, Mass., Tech. Rept. No. 262; May 1, 1957.

[3] R. Plonsey, "Diffraction by cylindrical reflectors," *Proc. IEE*, vol. 105C, pp. 312–317; March, 1958.

Lorentz reciprocity theorem,[4] namely,

$$\int_v E_2 \cdot J_1 dv = \int_v E_1 \cdot J_2 dv \tag{4}$$

where J_1 is the true source that sets up the electric field E_1 while J_2 sets up E_2. For condition 1 we consider a unipole fed by a coaxial line over an infinite ground plane. The true source is a current sheet located in the coaxial line far from the probe or the load. The current sheet is chosen as

$$J_r(r) = \frac{A}{r} a_r$$

and a match toward the load is assumed. With this excitation a current distribution $J_1(z)$ is set up on the probe, as shown in Fig. 2.

For condition 2 the primary source is considered to be a current element J_e at a large distance from the probe. Under these conditions a current $J_2(z)$ will be induced on the probe in the presence of the plane reflector. Furthermore an image source J_e^i and also an image probe source $J_2^i(z)$ appear, where $J_e = J_e^i$ and $J_2(z) = J_2^i(z)$ (see Fig. 3).

Our problem can now be formulated in terms of the aforementioned sources. $(J_e + J_e^i)$ taken together produce the field that exists in the absence of the probe detector while $J_2^i(z)$ modifies this field and is the source of the interaction error. Specifically we wish to compare the received signal due to $(J_e + J_e^i)$ with that due to $J_2^i(z)$.

The application of reciprocity is relatively straightforward and involves the assumption that the currents $J_1(z)$ and $J_2(z)$ may be taken as concentrated along the probe axis.[5] A further assumption is made that if E_i is the field set up at the probe by the currents $J_e + J_e^i$ then the electric field due to $J_2(z)$ just cancels E_i over the probe surface. Although this neglects the contribution to the field at the probe from its image it is consistent with our requirement that the latter effect be small.

The final result is an expression for the received signal, V_r, at the terminal end of the coaxial line due to the probe image current $J_2^i(z)$, and due to the primary signal source and its image $J_e + J_e^i$. The error introduced by the interaction of probe and reflector must then be less than or equal to the magnitude of the former quantity divided by the latter. We get

per cent error in V_r

$$= \frac{\left| \int_l^l \int_{-l}^l G(z - z', b) J_1(z') J_2(z) dz' dz \right|}{\int_{-l}^l \int_{-l}^l G(\xi - z', a) J_1(z) J_2(z') dz' dz} \times 100 \tag{5}$$

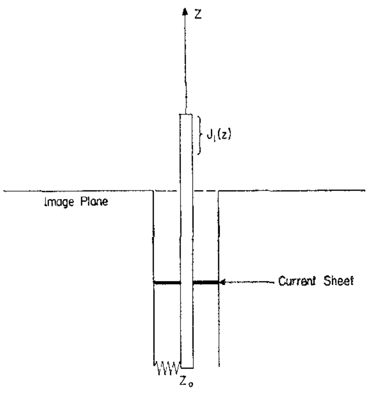

Fig. 2—Current source (condition 1).

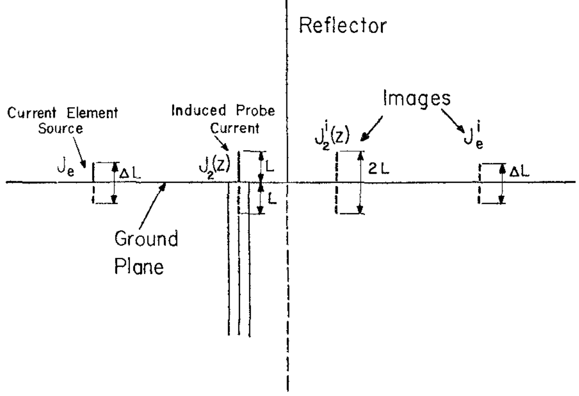

Fig. 3—Current source (condition 2).

where

$$G(\xi - z', a) = \frac{\mu}{4\pi}\left(-j\omega + \frac{1}{j\omega\mu\epsilon}\frac{\partial^2}{\partial \xi^2}\right)\frac{e^{-jkr'}}{r'}$$

$$G(z - z', b) = \frac{\mu}{4\pi}\left(-j\omega + \frac{1}{j\omega\mu\epsilon}\frac{\partial^2}{\partial z^2}\right)\frac{e^{-jkr}}{r}$$

and

$$r' = \sqrt{a^2 + (\xi - z')^2}$$
$$r = \sqrt{b^2 + (z - z')^2}.$$

The variables ξ, z, and z' can be identified from Fig. 4. Note that the result is independent of the magnitude of J_1 and J_2, nor does it depend on I_2, as is proper.

The problem considered here is related to one dealt with by Justice and Rumsey.[6] Their case differs in that a passive scatterer is considered, however the general approach is similar. In their curves for interaction error, however, the effect of probe radius is curiously absent.

A portion of a report by Hsu[7] is also devoted to a study of this problem. He uses a different measure of

[4] See for example R. Plonsey and R. Collins, "Principles and Applications of Electromagnetic Fields," McGraw-Hill Book Co., Inc., New York, N. Y., p. 420; 1961.

[5] In this application the reflector is ignored and the currents which make up J_2 are defined as the $(J_e + J_e^i + J_2^i)$ of Fig. 3.

[6] R. Justice and V. H. Rumsey, "Measurement of electric field distributions." IRE Trans. on Antennas and Propagation, vol. AP-3, pp. 177–180; October, 1955.

[7] H. P. Hsu, "Analysis of the Interaction Between a Measuring Probe and a Reflector," Case Inst. Tech., Cleveland, Ohio, Sci. Rept. No. 22 (AF 19(604)-3887); February, 1961.

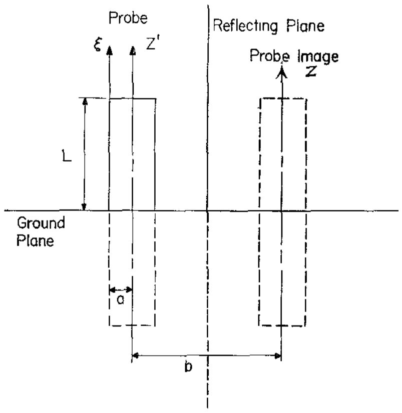

Fig. 4—Probe and image geometry.

interaction which is less clearly related to the received signal itself. His results appears to be too conservative.

Evaluation of Interaction

In evaluating (5) for specific probe-reflector geometry it is necessary to specify the functional form of the currents J_1 and J_2. We have taken

$$J_2 = \left(\frac{\cos kz - \cos kl}{1 - \cos kl}\right)$$

since, when acting alone, it results in a fairly uniform field over the central portion of the unipole probe. In this connection $\xi=0$ was chosen when evaluating the denominator of (5). The choice for J_1 appears to be less critical, and since it simplifies the calculation, $J_1 = \sin k(l-|z|)$ was used.

With these assumed values of $J_1(z)$ and $J_2(z)$ (5) can now be evaluated. The details follow very closely the mathematical treatment in Storer,[8] and the results are expressed in generalized sine and cosine integrals. Using the notation of King[9] we get

$$|\text{per cent error}| = \frac{[C_b(l,l) - C_b(l,0)] - \cos kl[E_b(l,l) - E_b(l,0)]}{(1-\cos kl)\left[2\sin kl\frac{e^{-jkR_1}}{kR_1} - \cos klE_a(l,0)\right]} \times 100$$

where $R_1 = \sqrt{a^2+l^2}$. The values of C_b, E_a, and E_b, are found from the Tables of Generalized Sine and Cosine Integral Functions[10] using the formula of King.[11] The per cent error was computed for $kl=0.39$ and $kl=0.60$ as a function of distance to the reflector with $ka=0.015$, 0.03, 0.05, 0.10. These results are plotted in Figs. 5 and 6.

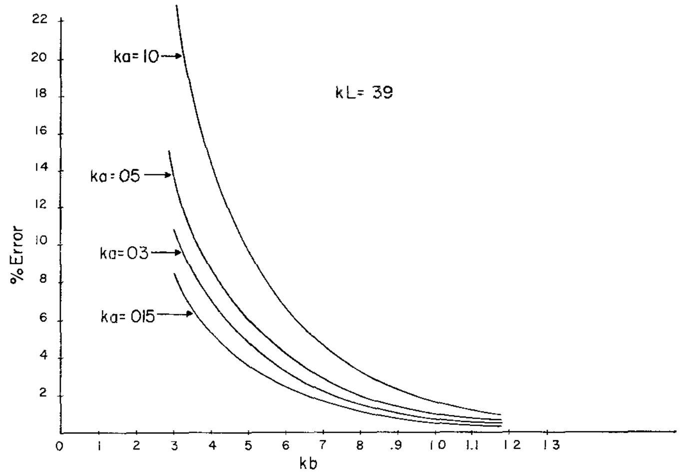

Fig. 5—Error due to interaction of probe of length $l=0.39/k$ with spacing to infinite conducting plane of $b/2$, for values of radius $a=0.015/k$, 0.03/k, 0.05/k, 0.10/k.

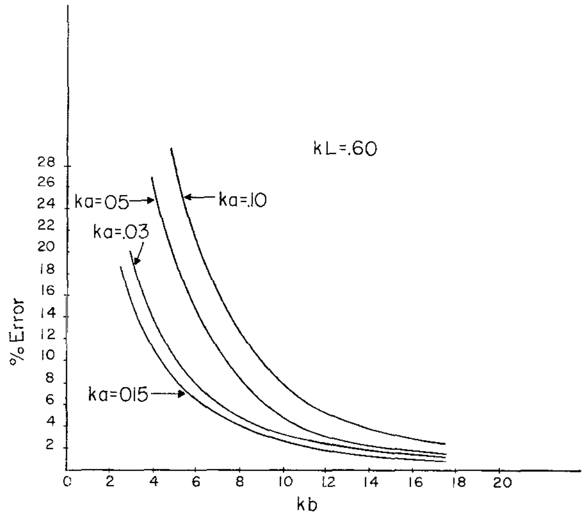

Fig. 6—Error due to interaction of probe of length $l=0.60/k$ with spacing to infinite conducting plane of $b/2$ for values of radius $a=0.015/k$, 0.03/k, 0.05/k, 0.10/k.

Experimental Program

Experimental verification of the types of error and overall performance of the electric-probe surface-current technique for a parallel-plane region was sought. For this purpose the X-band parallel-plane device at the Case Institute of Technology was used. Physical details of the device are given in a paper by Hsu.[12]

The probe used was a slightly modified Hewlett Packard (HP 444A) model. The modification consisted of the addition to the standard probe tip of a sleeve carrying a 0.006 inch diameter wire for subsequent thin-probe experiments. The latter probe protruded through a 0.020 inch hole in the parallel-plate device.

Data was taken using a flat strip reflector whose width was 8 inches and which was set perpendicularly between

[8] J. E. Storer, "Variational Solution to the Problem of the Symmetrical Cylindrical Antenna," Cruft Lab., Harvard University, Cambridge, Mass., Tech. Rept. No. 101; February 10, 1950.

[9] R. W. P. King, "The Theory of Linear Antennas," Harvard University Press, Cambridge, Mass., p. 94; 1956.

[10] Staff of the Computation Laboratory of Harvard University, "Tables of Generalized Sine and Cosine Integral Functions," Harvard University Press, Cambridge, Mass.; 1949.

[11] *Ibid.*, pp. 95, 96.

[12] H. P. Hsu, "Aperture fields in the diffraction by a slit," *J. Appl. Phys.*, vol. 31, pp. 1742–1746; October, 1960.

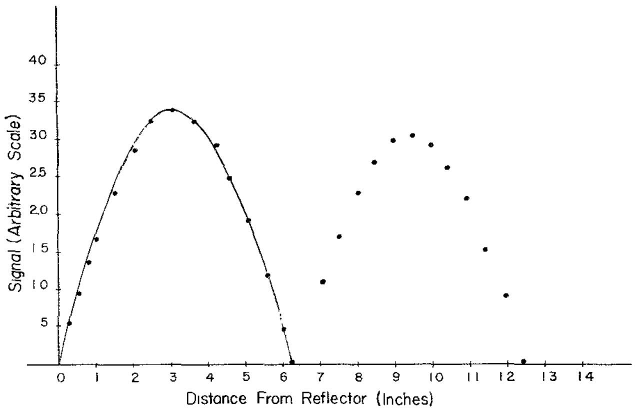

Fig. 7—Electric field near strip reflector.

the parallel plates (thus simulating an infinite strip or ribbon). The electric field was measured along the perpendicular bisector of the strip and a typical result is shown in Fig. 7 ($ka=0.015$, $kl=0.12$). The source here consisted of a linear probe (equivalent line source) along the perpendicular bisector of the reflector and at a distance of 27.75 inches. The distance between nulls of the measured field corresponds to a free-space wavelength (as computed from the measured frequency). Of great interest is the fact that a superposed sine curve with the same peak magnitude and period fits the data very closely, (except near the nulls), as shown in Fig. 7. As a consequence, rather than evaluate a rigorous solution of this problem, it seemed clear that very close to the reflector the field behavior could be taken as sinusoidal. (This would be exactly true if the reflector were of infinite extent, of course.)

The results of measurements along a perpendicular bisector for values of $kl \approx 0.39$ and 0.62, with $ka \approx 0.015$, are given in Table I and Table II. Also given are values obtained from an assumed sinusoidal variation, a computed experimentally determined error, and the error as obtained from the theory. The latter turns out to be greater than that found from the experimental work. However the measurements shown in Table I and Table II are subject to limitations in accuracy due to the particular device used. The main source of difficulty arises from the fact that the probe position is determined from angular information such that the accuracy is limited to ± 0.002 inch. Since the reflector position is also established with this same tolerance, there is a substantial uncertainty in small probe-reflector spacings. Consequently the experimental evaluation of the errors inherent in the probe technique is only semi-quantitative. Nevertheless the measurements and theory are not too far apart and indicate roughly the same transition point from large to small per cent error.

In any event the results clearly show that a generally acceptable level of performance is achievable with an electric probe at small distances from a reflector. A number of actual measurements of surface currents for flat and circularly curved strips but with a standard probe, were obtained by O'Flynn[13] and also appear satisfactory.

TABLE I

$ka=0.015$, $kl=0.39$, frequency $=9355$ Mc, $\lambda=1.262$ in.

Probe-reflector Distance (fraction of λ)	Measured field (relative value)	Fitted Sine Curve	Per Cent Deviation From Sine Curve	Per Cent Error From Fig. 5
0.0095	1.00	0.71	48	
0.019	1.50	1.44	4.0	
0.029	2.19	2.17	1.0	6.5
0.041	3.73	3.60	3.5	3.5
0.069	4.96	4.97	0.2	1.7
0.088	6.24	6.25	0.2	0.4
0.128	8.61	8.56	0.6	0
0.167	10.1	10.3	1.9	0
0.206	11.5	11.4	0.9	0
0.245	11.9	11.9	0.0	0
0.285	11.6	11.6	0.0	0
0.324	10.6	10.6	0.0	0
0.364	9.0	9.0	0.0	0
0 402	6.85	6.85	0.0	0
0.442	4.42	4.22	4.7	0
0.481	1.44	1.37	4.8	0
0.500	0	0	0	0

TABLE II

$ka=0.015$, $kl=0.62$, frequency $=9355$ Mc, $\lambda=1.262$ in.

Probe-reflector Distance (fraction of λ)	Measured field (relative value)	Fitted Sine Curve	Percent Deviation From Sine Curve	Percent Error From Fig. 6
0.0095	0.92	0.52	77	
0.019	1.30	1.05	24	
0.029	1.67	1.58	5.7	
0.041	2.79	2.63	6.1	8.0
0.069	3.76	3.64	3.3	3.0
0.088	4.62	4.56	1.3	2.2
0.128	6.31	6.25	1.0	1.0
0.167	7.66	7.56	1.3	0
0.206	8.51	8.37	1.7	0
0.245	8.70	8.70	0.0	0
0.285	8.41	8.48	0.8	0
0.324	7.68	7.77	0.0	0
0.364	6.39	6.57	0.3	0
0.402	4.84	5.00	2.0	0
0.442	3.03	3.09	1.0	0
0.481	0.91	1.00	4.0	0
0.500	0	0	0	0

Conclusion

It is apparent from the data and analysis that good measurements of reflector surface currents through the use of an electric probe can be achieved. The requirement that the probe-reflector distance not exceeds say, $\lambda/20$ is compatible with the requirement of small interaction. For example at X band a probe with the characteristic $ka=0.03$ and $kl=0.39$ operates satisfactorily. Such measurements as may be made with these probes should be quite useful in diffraction studies.

[13] Staff Rept., "Study of Control of Aperture Fields," Case Inst. of Tech., Cleveland, Ohio, Sci. Rept. No. 20, (AF (604)-3887) (The Study of Surface Current Measurements, as reported, was performed by M. O'Flynn.) Also M. O'Flynn, "Surface Current Measurements in a Parallel-Plate Region," M.S. thesis, Case Inst. of Tech., Cleveland, Ohio; 1961.